Towards a Mechanism for Ensuring Equity Over Time Among Airspace Users in Collaborative Flight Prioritization

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***Abstract*—In air traffic flow management (ATFM), the EUROCONTROL Network Manager regulates flights in the case of unexpected events that temporarily reduce airport capacity. In this case, regulated flights receive a new target time of arrival (TTA) following a first-planned, first-served approach. These regulations typically result in flight delays and additional delay costs for airspace users (AUs) and the arrival airport. The HARMONIC project, which is funded by the SESAR Joint Undertaking within the EU Horizon Europe program, develops, among others, a Target Time Management System (TTMS) to enable collaborative flight prioritization and optimization based on preferences of the AUs and the airport. The TTMS was successfully tested in live trials at Zurich Airport, with results indicating increased arrival punctuality and fewer missed connections. Although short-term inequity is acceptable to improve overall efficiency in a given situation, the TTMS shall ensure long-term equity, i.e., no AU is favored or disfavored compared to the others over time. In this regard, one challenge is to design an Equity Mechanism as part of the TTMS to ensure long-term equity across AUs. In this paper, we introduce an Equity Mechanism that modifies the preferences of AUs. We investigate various configurations of three different strategies for adjusting the preferences of AUs with the Equity Mechanism. We use the Theil index to measure inequity and preferences from 51 regulations obtained during the live trials at Zurich Airport for re-running each optimization in a controlled lab environment. Our results show that some configurations of the Equity Mechanism can reduce the Theil index over time, i.e., successfully improve long-term equity, while having only a minor impact on the quality of the flight lists found by the TTMS.**

***Index Terms*—air traffic control, flight prioritization, equity**

I. INTRODUCTION

Managing the air traffic flow can be challenging, especially under capacity reductions, frequently caused by, for example, worsening weather conditions or other incidents. Under a demand-capacity imbalance in Air Traffic Flow Management (ATFM), EUROCONTROL's Network Manager issues a regulation for the congested airports, which typically results in slots assigned to flights become unavailable. With a regulation in effect, regulated arrival flights are assigned new calculated take-off times at the departure airport, which potentially leads to a new target time of arrival (TTA) to be assigned to each regulated flight, commonly assigned following the principle of *first-planned, first-served* (FPFS) [1]. The FPFS allocation of ATFM slots generally preserves the original order of the flights but shifts the flights back in time. Consequently, the FPFS allocation typically yields evenly distributed flight delays for each airspace user (AU) while equity is maintained through equal treatment of all flights [1], [2].

The FPFS process is well-established and acknowledged by AUs, but it does not account for delay costs of flights. Delay costs of flights typically differ and entail, for example, passenger-related costs, e.g., connection and baggage costs, or operational costs, e.g., crew duty limit violations [3], [4]. To give AUs greater flexibility to prioritize regulated flights and realize cost savings, the User-Driven Prioritization Process (UDPP) was established, allowing intra-airline ATFM slot swapping [1]. Although equity is preserved in the UDPP as internal exchanging of ATFM slots cannot negatively impact other AUs [1], opportunities for greater cost savings through collaborative flight prioritization across AUs are not captured. These opportunities were investigated in the context of different research projects.

The exploratory research project SlotMachine [5], funded by the SESAR Joint Undertaking under the European Union's Horizon 2020 research and innovation program, developed a platform that optimizes flight lists based on priority inputs submitted by different AUs. As these priority inputs of flights capture AUs' preferences for certain slots, they contain confidential information from which a flight's cost structure could be derived. To protect confidentiality and prevent information leakage of preferences, the SlotMachine platform uses multi-party computation (MPC) with secret-sharing [6]. In addition to MPC, the system comprises an auction-based market mechanism [3] that allows AUs to bid on ATFM slots an AU has offered. Bids can be placed using non-monetary delay credits that are only valid within the system and which can only be earned by accepting additional delays.

The industrial research project HARMONIC [7], funded by the SESAR Joint Undertaking within the EU Horizon Europe program, continues SlotMachine's work on local flight list optimization. In HARMONIC, a Target Time Management System (TTMS) was developed by the project partners Zurich Airport, Swiss International Airlines (SWISS), Frequentis AG, Skyguide, AIT Austrian Institute of Technology, and the Institute of Business Informatics – Data & Knowledge Engineering at Johannes Kepler University Linz. The TTMS extends parts of the optimization platform created during the SlotMachine project and enables collaborative optimization of flight lists based on the preferences of the arrival airport and the AUs. The TTMS aims to improve overall efficiency in the slot allocation while reducing delay costs for both the airport and the AUs.

With the TTMS in place, each AU participating in a regulation can submit preferences for feasible combinations of TTAs for its flights, while the arrival airport submits preferences for the TTAs for all regulated flights. These preferences are used to find optimized flight lists and are expressed as *weight maps*. A weight map can be interpreted as a matrix containing numerical values that represent the priority of a flight for a certain TTA, with higher values indicating higher priorities. With Zurich Airport acting as trusted platform provider [8], the TTMS was successfully tested in initial live trials in May and June 2025; we refer to Hagemann et al. [4] for a presentation of the results.

The weight maps submitted by the AUs for optimization must comply with a set of predefined conformance criteria designed to prevent individual preferences from intentionally manipulating the optimization process [9]. These conformance criteria therefore aim to promote fairness among AUs when optimizing flight lists collaboratively. However, fairness does not necessarily imply equity, and the optimization component of the TTMS may still favor certain AUs in the long run. As a result, delay distributions may become imbalanced, with some AUs receiving more delay than others. Although some short-term inequity is necessary and accepted by AUs to enhance slot allocation efficiency, the delay allocation should balance out over time, with all AUs receiving approximately equal delays. Whereas SlotMachine's platform ensures equity across AUs over time via an auction-based market mechanism, HARMONIC's TTMS adapts the idea of *inequity weights* [10] with the goal of long-term equity. To this end, an Equity Mechanism is developed as part of the TTMS that adjusts AUs' weight maps to prevent any AU from being favored or disfavored compared to others over time.

In this paper, we introduce our Equity Mechanism and investigate three different strategies—namely, *multiplication*, *softmax-based*, and *exponential decay*—for adjusting AUs' weight maps to improve equity over time. In a controlled lab environment at Frequentis Headquarters in Vienna in September 2025, we used the weight maps from 51 regulations obtained during the live trials at Zurich Airport and tested a total of 120 configurations across the three different strategies to calculate inequity weights. Regardless of the strategy, the Theil index is used as inequity measure, as proposed by Carré [10], with each AU's contribution to the index serving as the basis for calculating its custom inequity weight.

For a given configuration, we re-ran each optimization and measured inequity over time based on the optimization results within a rolling window covering the prior 20 regulation instances. Our results show that certain strategies, in combination with appropriate configurations, are able to considerably lower the Theil index over time, leading to an increase of equity across AUs in the long run. Furthermore, with the Equity Mechanism in place, only a minor impact on the quality of the flight lists found by the TTMS can be observed.

The remainder of this paper is structured as follows. Section II provides background information on the TTMS developed in the HARMONIC project, on flight prioritization, particularly with focus on equity and fairness concepts, and on different indices to measure equity. Section III introduces the Equity Mechanism itself and explains how inequity weights are calculated under a specific strategy. Section IV presents the experimental setup used during the validation exercise in Vienna as part of the HARMONIC project. Section V describes the experimental results of the Vienna validation exercise. Section VI concludes the paper with a summary of the main findings and an outlook on future work.

II. BACKGROUND

In the following, we provide an overview of the TTMS architecture developed in the HARMONIC project, concisely discuss equity and fairness in the context of flight prioritization, and provide an overview of different equity measures.

A. Target Time Management System

To establish a platform that allows for collaborative optimization of arrival flight lists, a *Target Time Management System* (TTMS) is developed as part of the industrial research project HARMONIC. Fig. 1 shows the TTMS architecture, with the Orchestrator, the Optimizer, and the Equity Mechanism as its core components.

During operations, the airport and the AUs continuously send their preferences, expressed as weight maps, to the Orchestrator, which stores them in a database. In case of a regulation, the Orchestrator requests necessary information from EUROCONTROL's Network Manager, which can be used to identify regulated flights and available TTAs. Subsequently, the Orchestrator selects from the database the relevant weight maps for the regulated flight, as provided by the airport and the AUs. A weight map is a matrix where each row corresponds to a flight and each column corresponds to a TTA, with the elements being numerical values that express the utility of a TTA for a flight—higher values denote a higher arrival priority for a flight at a certain TTA. The airport provides a weight map covering all regulated flights, while each participating AU provides a weight map for its regulated flights.

Individual AUs may construct their weight map based on flight-specific delay cost profiles that account for aspects such as connection and baggage costs, crew duty limit violations, or noise charges [4]. Given the non-linear increase in cost over time the further a flight is delayed, a flight's individual cost profile can be represented as a step function with discrete jumps when delay thresholds are exceeded [6]. These cost profiles then need to be mapped into a weight map, for example, using a quadratic cost inversion, as implemented by SWISS during the live trials at Zurich Airport in May and June 2025; we refer to Hagemann et al. [4] for more details on the cost profiles.

Once the TTMS Orchestrator receives all data required for optimization, it checks whether equity among AUs over time is given. If the measured inequity exceeds a certain threshold, the Orchestrator triggers an inequity weight calculation and transmits all relevant AUs' weight maps to the Equity Mechanism. For each AU, the Equity Mechanism computes its specific inequity weight and applies it to that AU's weight map. The equity-adjusted preferences are sent back to the Orchestrator, which triggers an optimization to find optimal arrival flight lists based on the AUs' and airport weight maps.

With a trusted TTMS provider, e.g., the airport, the Optimizer uses a deterministic algorithm, e.g., the Hungarian Method, to assign regulated flights to available TTAs optimally [8]. For optimization, the airport weight map and AUs' weight maps are normalized and combined at an equal ratio, and, since weights reflect utility, the Optimizer's objective is to maximize the sum of weights. Given the resulting assignment solution, a fitness value is computed for the airport and the AUs by summing the weights of the assigned flight-TTA pairs in their corresponding weight map. We refer to Gruber et al. [8] for more information on the case where the TTMS is hosted by an honest-but-curious third-party platform provider.

Following optimization, the determined arrival sequence is sent to the Orchestrator, which evaluates the flight list against predefined safeguards to ensure that, for example, the ATFM delay caused by TTMS does not exceed the delay under a conventional regulation. Following verification of compliance with all safeguards, the optimized flight list may be transmitted to EUROCONTROL's Network Manager for potential integration into the system for execution.

B. Equity and Fairness in Flight Prioritization

Equity and fairness are crucial aspects in the context of ATFM, particularly in the allocation of ATFM slots when demand exceeds capacity. Several central questions arise in this context, including how priority should be assigned to aircraft, what constitutes equity, and how available resources can be used efficiently [11]. As demand-capacity imbalances limit available resources and causes flight delays, the resulting delays should be distributed as equitably as possible [11]. Traditionally, flight delays are allocated according the *first-planned, first-served* (FPFS) principle, which assigns ATFM slots such that the original sequence of flights is largely preserved [1], [2]. By applying delays in order of the scheduled flight sequence, the FPFS principle does not distinguish between flights, thus maintaining equity and fairness among AUs [1], [2]. However, although well-established and generally accepted by AUs, the FPFS process does not account for the preferences of the individual AUs.

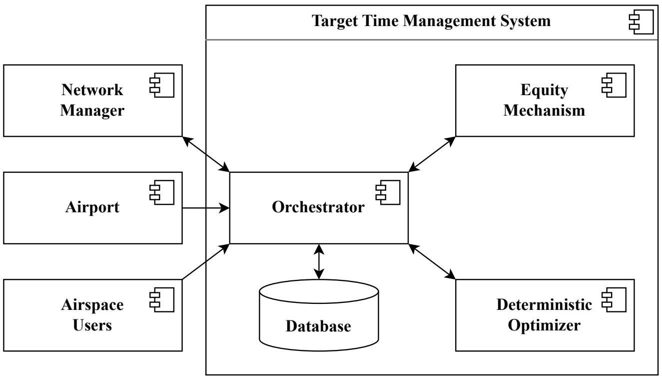

Fig. 1. Architecture of the Target Time Management System

The *User-Driven Prioritization Process* (UDPP) allows AUs to prioritize their flights [1]. The UDPP gives the AUs the flexibility to swap the assigned ATFM slots internally, thereby allowing them to prioritize their flights based on their business needs [1]. Equity and fairness among AUs are preserved by ensuring that internal swaps of ATFM slots do not influence flights of AUs not participating in the UDPP [1].

In HARMONIC's predecessor project, SlotMachine, a platform was developed that enables inter-airline ATFM slot swapping through an auction-based market mechanism [3]. The mechanism allows each participating AU to prioritize its flights by offering its allocated ATFM slots and bidding on desired slots offered by other AUs [3]. By using non-monetary delay credits for bidding, which can only be earned by accepting delays, the proposed market-mechanism aims to ensure equity over time and fairness among AUs [3].

In contrast to SlotMachine, the TTMS developed in HARMONIC does not use an auction-based market mechanism. Instead, HARMONIC's TTMS adapts the concept of inequity weights [10] to ensure equity over time among AUs and employs conformance criteria [9] to promote fairness in the optimization preferences of AUs. For TTMS optimizations, AUs submit preferences—expressed as weight maps—that must adhere to predefined conformance criteria, which aim to prevent intentional manipulation of the process to gain an unfair advantage over other AUs [9]. To ensure fairness in the live trials at Zurich Airport in May and June 2025, a maximum value per weight map within a regulation was applied, and each flight's average assigned weight was required to meet a defined threshold [4]. In this paper, we investigate different strategies to calculate inequity weights for AUs that modify their weight maps to promote long-term equity.

C. Measuring Equity

Selecting an appropriate measure to assess equity within a domain can be challenging, as it involves making assumptions and decisions about distributional characteristics that inevitably shape the conclusions drawn from the results [12], [13]. Consequently, there is no single best measure and the choice should be tailored by the goals of the analysis [12]. Different approaches, for example the axiomatic approach, which imposes a set of principles that are generally desirable for an equity measure, can assist in selecting an appropriate measure for a particular use case [12]. Across a broad range of domains, established equity indices include the Gini index, the Atkinson index, and the Theil index [12], [13].

The Gini index is a scale-invariant measure bounded between 0 (perfect equity) and 1 (perfect inequity) [14]. Originally developed to understand income distributions, the Gini index is now a commonly applied inequity measure across domains, presumably due to its simplicity and broad acceptance [14]. As a limitation, the Gini index does not directly reflect individuals' perception of inequity, and, when used without additional indicators, it offers only a limited view in inequity within a distribution [14].

The Atkinson index aggregates individual welfare using an explicit social welfare function combined with an inequity-aversion parameter that controls the sensitivity to disparities within the distribution [12]. Like the Gini index, the Atkinson index takes a value of 0 under perfect equity and increases as dispersion within a distribution grows [13]. Because the weighting across different parts of the distribution is governed by the chosen inequity-aversion parameter, the Atkinson index and any conclusions drawn from it are shaped by the judgment about how strongly inequity should be penalized [12], [13].

The Theil index is an entropy-based inequity measure grounded in information theory [12], [15]. The Theil index ranges from 0 under perfect equity to a maximum equal to the natural logarithm of the number of individuals in the population, so its potential maximum increases with population size [15], [16]. Due to its additive decomposition property, the Theil index can be used to compare inequity between and within different subgroups of the population, while being more sensitive to changes at the lower end of the distribution [12], [15]. However, by convention, if an individual's value of interest in the population is zero, its contribution to the Theil index equals zero, despite the natural logarithm of zero being undefined [15].

In air traffic management (ATM), Carré [10] examined and evaluated the Gini, Atkinson, and Theil indices against several requirements, including decomposability to determine airline-specific inequity contributions and calculability over a defined history of instances. The research by Carré [10] showed that the Theil index can be used in in the context of ATM applications. Consequently, in this paper and for the current series of experiments, the HARMONIC project chose to use the Theil index to assess equity among AUs.

III. EQUITY MECHANISM

In this section, we first introduce the basis for calculating the inequity weight for an AU and subsequently describe the three different strategies to calculate inequity weights—namely, *multiplication*, *softmax-based*, and *exponential decay*. The project adopted the Theil index as inequity measure between AUs and the original idea of inequity weights to adjust the preferences, i.e., weights, submitted by AUs, following research by Carré [10].

A. Equity Contribution

The basis for calculating the inequity weight for an AU is its inequity contribution to the Theil index. To calculate the contribution of each AU to the Theil index, the AU's average delay and the average delay of all AUs are required.

Initially, the delay for each flight x is calculated, regardless of the AU. Let s_x be the last TTA assigned by the Optimizer of the TTMS and τ_x be the preferred time wished for the respective flight x, the delay of a flight δ_x can then be calculated as follows.

$$\delta_x = |s_x - \tau_x| \tag{1}$$

To determine the average delay of an AU, the calculated delays of the AU's flights are used. Let F_a denote the set of flights of an AU a, the average delay μ_a of the AU can then be calculated as follows.

$$\mu_a = \frac{1}{|F_a|} \times \sum_{x \in F_a} \delta_x \tag{2}$$

To determine the average delay of all AUs, the calculated delays of all flights are used. Let F denote the set of all flights, regardless of the AU, the average delay μ over all AUs can then be calculated as follows.

$$\mu = \frac{1}{|F|} \times \sum_{x \in F} \delta_x \tag{3}$$

The average delay of an AU and the average delay of all AUs are then used to determine an AU's contribution to the Theil index. Let μ_a be the average delay of an AU a and μ be the average delay of all AUs, the contribution to the Theil index c_a of an AU can then be calculated follows.

$$c_a = \frac{\mu_a}{\mu} \times ln\left(\frac{\mu_a}{\mu}\right) \tag{4}$$

The value of c_a is influenced by whether an AU a was advantaged or disadvantaged. If an AU was advantaged ($\mu_a < \mu$), c_a becomes negative ($c_a < 0$). Conversely, if an AU was disadvantaged ($\mu_a > \mu$), c_a becomes positive ($c_a > 0$). If an AU was neither advantaged nor disadvantaged ($\mu_a = \mu$), c_a becomes zero. A special case arises if an AU was advantaged and had an average delay of zero. If $\mu_a = 0$, the Theil contribution c_a of an AU a cannot be calculated as defined in Equation 4, because $ln(0)$ is undefined. Therefore, by convention, c_a also becomes zero.

To determine the Theil index, the individual inequity contributions of the AUs are used. Based on the AU-specific c_a, the Theil index TH can be calculated as follows, where A is the set of all AUs participating in the optimization.

$$TH = \frac{1}{|A|} \times \sum_{a \in A} c_a \tag{5}$$

B. Inequity Weight Calculation Strategies

The strategies to calculate inequity weights are applied with the aim of increasing equity among AUs over time. Each strategy is designed to counteract potential inequities that may arise when collaboratively optimizing flight lists. In the following, the three strategies implemented—namely, *multiplication*, *softmax-based*, and *exponential decay*—to calculate inequity weights are introduced. Furthermore, it is described how the calculated inequity weights modify the preferences, i.e., weights, submitted by the individual AUs.

The calculation of the inequity weights can be configured to only benefit disadvantaged AUs. When this option is used, the inequity weights are only calculated for AUs that have a higher average delay than the average delay of all AUs. In addition, the adjustment of the AU's flight preferences can be configured to allow a weight decrease below zero. When this option is used, the adjusted weights of an AU's flight can become negative when applying the AU's inequity weight.

For an optimization, each AU defines a weight map W expressing the AU's preferences regarding the optimization. The weight map is a matrix $W = (w_{xy})$, where w_{xy} is the weight representing the utility of flight x for TTA y. Consider the following example, which serves as a running example to illustrate the different calculation mechanisms for inequity weights throughout the remainder of this section. There are two flights of two distinct AUs, each with an initial weight map $W_a := [1\,000, 800, 400, 200, 50]$, an average delay delay over all AUs $\mu = 10$, and the following average delay for each individual AU.

- AU 1: $\mu_1 = 15$ (disadvantaged because $\mu_1 > \mu$)
- AU 2: $\mu_2 = 5$ (advantaged because $\mu_2 < \mu$)

Based on the average delay μ_a of each AU a and the average delay μ of all AUs, the resulting Theil index TH is 0.13. The corresponding contributions are $c_1 = 0.61$ for AU 1 and $c_2 = -0.35$ for AU 2.

1) Multiplication: The *multiplication* strategy to calculate inequity weights multiplies the Theil contribution c_a of an AU a with a constant factor m. The inequity weight p_a of an AU is calculated as follows.

$$p_a = c_a \times m \tag{6}$$

The calculated inequity weight for an AU is then applied to adjust its flight preferences, i.e., weights. For each AU a, let $W_a = (w_{xy})$ be the AU's weight map and p_a be the AU's calculated inequity weight, each weight w_{xy} in W_a that satisfies $w_{xy} \geq 0$ is then modified as follows.

$$w_{xy} := w_{xy} + p_a \tag{7}$$

The multiplication strategy can be described as easy to understand and straightforward, as only a constant factor is needed to calculate inequity weights. In the running example, if $m = 100$, the modified weight maps are $W_1 := [1\,061, 861, 461, 261, 111]$ for AU 1 (disadvantaged) and $W_2 := [965, 765, 365, 165, 15]$ for AU 2 (advantaged).

2) Softmax-Based: The *softmax-based* strategy to calculate inequity weights applies the softmax function with temperature T—the higher the temperature, the more uniform the distribution of the inequity weights—to the Theil contribution c_a of an AU a. The inequity weight p_a for an AU is calculated as follows, where A is the set of all AUs participating in the optimization.

$$p_a = \frac{e^{\frac{c_a}{T}}}{\sum_{a \in A} e^{\frac{c_a}{T}}} \tag{8}$$

The calculated inequity weight for an AU is then applied to adjust its flight preferences, i.e., weights. For each AU a, let $W_a = (w_{xy})$ be the AU's weight map, c_a be the AU's Theil contribution, and p_a the AU's calculated inequity weight, and let sgn be the signum function that returns 1, 0, or -1 based on the value of c_a, each weight w_{xy} in W_a that satisfies $w_{xy} \geq 0$ is then modified as follows.

$$w_{xy} := w_{xy} + \operatorname{sgn}(c_a) \times w_{xy} \times p_a \tag{9}$$

The softmax-based strategy benefits from mapping the inequity contributions into action probabilities, whereby the resulting distribution can be controlled using a temperature. In the running example, if $T = 1$, the adjusted weight maps are $W_1 := [1\,722, 1\,378, 689, 344, 86]$ for AU 1 (disadvantaged) and $W_2 := [722, 578, 289, 144, 36]$ for AU 2 (advantaged).

3) Exponential Decay: The *exponential decay* strategy applies the exponential function to calculate an individual inequity weight for each weight of a flight of an AU while considering the number of non-negative weights that an AU has indicated for a flight. Let W_a be the weight map of an AU a, and let $W_a^x = (w_y)$ be the weights that the AU has indicated for a particular flight x, i.e., the row in the weight map W_a that corresponds to flight x. Let $W_{a,\geq 0}^x$ denote the vector obtained from W_a^x by removing all negative components. Let $|W_{a,\geq 0}^x|$ be the number of non-negative weights in W_a^x and let k_{xy} be the position of w_y in W_a^x, the positional factor t_{xy} for a weight w_y in $W_{a,\geq 0}^x$ can then be calculated as

$$t_{xy} = |W_{a,\geq 0}^x| - k_{xy} + 1 \tag{10}$$

such that the first weight w_y in $W_{a,\geq 0}^x$ has $t_{xy} = |W_{a,\geq 0}^x|$ and the last weight w_y in $W_{a,\geq 0}^x$ has $t_{xy} = 1$.

The exponential decay strategy penalizes early-position weights more, as these positions typically carry the highest weights. In contrast to the multiplication and softmax-based strategy, the exponential decay strategy offers two approaches for calculating the specific inequity weights of flights of an AU, taking into account a regularization term λ.

According to the first approach, given a weight map $W_a = (w_{xy})$ of an AU a, the specific inequity weight p_{xy} for a non-negative weight w_{xy} in W_a is calculated using the positional factor t_{xy}, the AU's pure Theil contribution c_a, and the regularization term λ as follows.

$$p_{xy} = e^{c_a \times t_{xy} \times \lambda} \tag{11}$$

According to the second approach, given a weight map $W_a = (w_{xy})$ of an AU a, the specific inequity weight p_{xy} for a non-negative weight w_{xy} in W_a is calculated using the positional factor t_{xy}, the AU's resulting share $\sigma(c_a)$ obtained by applying the softmax function with temperature T to the Theil contribution c_a, and the regularization term λ along with the signum function sgn that returns 1, 0, or -1 based on the value of c_a as follows.

$$p_{xy} = e^{\mathrm{sgn}(c_a) \times \sigma(c_a) \times t_{xy} \times \lambda} \tag{12}$$

Note that $\sigma(c_a)$ is calculated as follows, where A is the set of all AUs participating in the optimization.

$$\sigma(c_a) = \frac{e^{\frac{c_a}{T}}}{\sum_{a \in A} e^{\frac{c_a}{T}}} \tag{13}$$

The calculated inequity weight for each weight of a flight for an AU—regardless of which approach is chosen—is then applied to adjust its flight preferences, i.e., weights. For each AU a, let $W_a = (w_{xy})$ be the AU's weight map and p_{xy} be the AU's calculated inequity weight for the weight w_{xy}, each non-negative weight w_{xy} in W_a is then modified as follows.

$$w_{xy} := w_{xy} \times p_{xy} \tag{14}$$

In the running example, if the first approach is used and $\lambda = 0.05$, the adjusted weight maps are $W_1 := [1\,164, 903, 438, 213, 52]$ for AU 1 (disadvantaged) and $W_2 := [917, 746, 380, 193, 49]$ for AU 2 (advantaged). If the second approach is used in combination with $\lambda = 0.05$ and $T = 1$, the adjusted weight maps are $W_1 := [1\,198, 924, 446, 215, 52]$ for AU 1 (disadvantaged) and $W_2 := [933, 757, 384, 195, 49]$ for AU 2 (advantaged).

IV. EXPERIMENTAL SETUP

This section describes the configurations of the three inequity weight calculation strategies analyzed during the HARMONIC validation exercise in Vienna in September 2025. In addition, it outlines the evaluation process to identify and further analyze the best configuration. For the validation exercise, we used the weight maps from 51 ATFM regulations for optimization, collected during the eight-week live trial (May and June 2025) at Zurich Airport.

TABLE I
PARAMETERS USED FOR THE MULTIPLICATION AND SOFTMAX-BASED STRATEGIES, AND FOR THE EXPONENTIAL DECAY STRATEGIES (WITH AND WITHOUT SOFTMAX) TO CALCULATE INEQUITY WEIGHTS

Strategy	**Parameters**
Multiplication	$m \in \{10^2, 10^3, 10^4, 10^5, 10^6\}$
Softmax-Based	$T \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$
Exponential Decay	$\lambda \in \{0.05, 0.10, 0.15, 0.20, 0.25\}$
Exponential Decay with Softmax	$\lambda \in \{0.05, 0.10, 0.15, 0.20, 0.25\}$, $T \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$

A. *Configurations*

A total of 120 configurations were tested across the three different strategies to calculate inequity weights. For each strategy, the analyzed configurations cover all meaningful combinations, considering options of only benefiting disadvantaged AUs and allowing weights to decrease below zero when applying inequity weights. Disadvantaged AUs are defined as those with an average delay higher than the average delay across all AUs (see Subsection III-A). When the setting to benefit only disadvantaged AUs is enabled, the modified weight cannot decrease below zero, because disadvantaged AUs receive an increase to their original weights through the applied inequity weight. In contrast, when inequity weights are calculated for both advantaged and disadvantaged AUs, the adjusted weights of advantaged AUs could decrease below zero. Table I shows the parameters tested specific to the respective strategy to calculate inequity weights (see Subsection III-B).

A total of 15 configurations were analyzed for both the multiplication and softmax-based strategies. For the multiplication strategy, five distinct constant factors were tested, and for the softmax-based strategy, five distinct temperatures were tested.

For the exponential decay strategy, we analyzed a total of 90 different configurations. Five distinct regularization terms were tested, regardless of whether the strategy used the pure Theil contribution or the resulting share obtained by applying the softmax function to the Theil contribution. When the exponential decay strategy was combined with the softmax function, the same temperature values as with the softmax-based strategy were tested.

B. *Evaluation Process*

The evaluation of each configuration of the individual strategies to calculate inequity weights is based on analyzing the Theil index over time. For comparison, we also computed the Theil index over time for the baseline. For the baseline, no inequity weights are calculated and applied, meaning that the true weight maps of the individual AUs collected during the live trials at Zurich Airport are used for optimization. In general, inequity weights for AUs are calculated if the Theil index over time is greater than or equal to a defined threshold. For the evaluation, this threshold was set to 0.001, as suggested by Carré [10]—findings from the live trials and the validation exercise indicate that this threshold is below achievable levels. To calculate the Theil index over time, we use a rolling window of the last 20 regulation instances.

The first Theil index is computed for the regulation instances 1–20 and is identical across all configurations and the baseline. For each AU, an inequity weight is calculated based on its contribution to the Theil index and subsequently applied to its weights submitted for the optimizations within the twenty-first regulation. From regulation instance 21 onward, inequity weights are applied and the optimization results are considered for all subsequent calculations. Consequently, the first Theil index, which relies solely on optimizing adjusted preferences, is computed for the regulation instances 21–40.

The process of calculating the Theil index over the last 20 regulation instances and applying inequity weights for the optimizations within the subsequent regulation is repeated until the last regulation, i.e., the fifty-first regulation, is reached. Overall, this results in 32 Theil index values for the baseline and each of the 120 configurations.

1) Performance: The Theil index values for the baseline and for each configuration of the inequity weight calculation strategies are used to evaluate their performance. To measure the performance, the Area under the Curve (AUC) is computed using the composite trapezoidal rule provided by the *numpy* library [17]. We assume that a lower AUC is better, as it indicates a reduction in the Theil index over time, which suggests an increase in equity among AUs over time. The AUC is used to identify the best configuration for each inequity weight calculation strategy—the one with the lowest AUC within that strategy–and to determine the overall best configuration—the one that minimizes the AUC.

2) Cost of Equity: The cost of equity measures how emphasizing equity affects the quality of the flight lists found by the TTMS. The cost of equity is determined for each optimization across the 31 regulations (regulation instances 21–51) in which inequity weights were calculated using the overall best configuration, i.e., the one that minimized the AUC. To compute the cost of equity, fitness values for the airport and the AUs are required. In general, fitness values are derived from the assignment solutions produced by optimization. Each assignment solution specifies flight-TTA pairs, which are used to calculate fitness values for the AUs and the airport by summing the weights in their corresponding weight map.

The fitness values derived from the baseline weight maps, i.e., the weight maps of the individual AUs and the airport collected during the live trials at Zurich Airport, serve as *ground truth* for the evaluation. For comparison, we optimize the airport weight maps combined with the equity-adjusted AUs' weight maps. Following optimization, new assignment solutions of flights to TTAs are derived that emphasize equity. These flight-TTA pairs are then combined with the baseline weight maps, i.e., those used during the live trials at Zurich Airport, to calculate another set of fitness values. We refer to these fitness values as *equity fitness values*.

Across optimizations, the ground truth and equity fitness values may differ in both scale and sign. To ensure comparability, we apply min-max normalization to the ground truth fitness f_i^* and equity fitness f_i^{eq} for each objective $i \in \{AUs, Airport\}$, rescaling values in the range $[0, 1]$. To determine the objective's minimum fitness f_i^{min} and maximum fitness f_i^{max}, we separately optimized each objective's weight map using a deterministic algorithm with minimization and maximization objective functions, respectively. The normalized ground truth fitness value $\tilde{f}_i^*$ and equity fitness value $\tilde{f}_i^{eq}$ for an objective i for an optimization can be calculated as follows.

$$\tilde{f}_i^* = \frac{f_i^* - f_i^{min}}{f_i^{max} - f_i^{min}}, \qquad \tilde{f}_i^{eq} = \frac{f_i^{eq} - f_i^{min}}{f_i^{max} - f_i^{min}} \tag{15}$$

We use the normalized fitness values to calculate the objective's cost of equity γ_i for an optimization as follows.

$$\gamma_i = \frac{\tilde{f}_i^{eq} - \tilde{f}_i^*}{\tilde{f}_i^*} \tag{16}$$

The objective's cost of equity can be positive, negative, or zero. The cost of equity is positive when $\tilde{f}_i^{eq} > \tilde{f}_i^*$, indicating a relative improvement of the objective's fitness. Conversely, the cost of equity is negative if $\tilde{f}_i^{eq} < \tilde{f}_i^*$, indicating a relative decline of the objective's fitness due to emphasizing equity. The cost of equity is zero if $\tilde{f}_i^{eq} = \tilde{f}_i^*$, indicating that considering equity did not affect the objective's fitness.

In general, the cost of equity is expected to be negative, because adding additional constraints, i.e., emphasizing equity, typically shifts the solution away from the unconstrained optimum. In a collaborative optimization setting, however, including inequity weights for AUs may tilt the result toward one objective, even when both objectives are considered equally for optimization. Therefore, since improving one objective necessarily worsens the other, a positive cost of equity for one objective implies a negative cost of equity for the other.

V. EXPERIMENTAL RESULTS

This section presents our results of the experimental evaluation of the performance of the different configurations for the inequity weight calculation strategies. In addition, we present the cost of equity results for the configuration that reduces the Theil index the most over time.

A. Performance

To assess the performance of each configuration for each strategy to calculate inequity weights, the AUC was calculated. The AUC was calculated based on the Theil index values determined using a rolling window of the last 20 regulation instances. In addition, the AUC was also calculated for the baseline, i.e., without considering equity. Figure 2 shows the progression of the Theil index over time for the baseline and for the configurations for the inequity weight strategies multiplication, softmax-based, and exponential decay. The baseline indicated an AUC of 19.06.

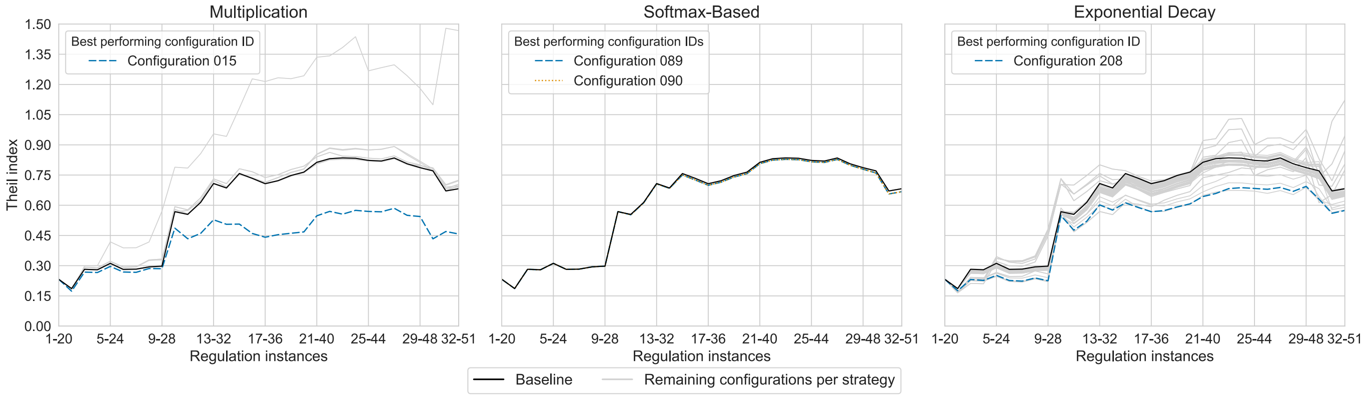

Fig. 2. Progression of the Theil index over time using a rolling window of the last 20 regulation instances for the baseline, i.e., without considering equity, and for each configuration analyzed for the three different strategies to calculate inequity weights. The best configuration(s) per strategy, i.e., the one(s) with the lowest AUC, are highlighted within the respective subplot

1) Multiplication: A total of 15 different configurations were analyzed for the multiplication strategy to calculate inequity weights, testing five different constant factors, i.e., 10^2, 10^3, 10^4, 10^5, and 10^6. The different configurations, which combine various constant factors with additional parameters, have varying effects on the Theil index over time and, therefore, on the AUC.

The overall highest AUC (29.72) was calculated for the configuration using a constant factor $m = 10^6$. For this configuration, inequity weights were calculated for both advantaged and disadvantaged AUs, and a weight decrease below zero when modifying the weights was allowed.

The best-performing configuration for the multiplication strategy (highlighted in the first column in Fig. 2) has an AUC of 13.63. This configuration also used a constant factor $m = 10^6$ and calculated inequity weights for both advantaged and disadvantaged AUs, but it disallowed a weight decrease below zero when modifying the weights. Thus, the best- and worst-performing configurations differ only in whether weights for advantaged AUs are allowed to decrease below zero when applying the inequity weights.

2) Softmax-Based: A total of 15 different configurations were analyzed for the softmax-based strategy to calculate inequity weights, testing five different temperatures, i.e., 0.2 to 1.0 in increments of 0.2, when applying the softmax function. Varying the temperatures for the softmax function applied to the Theil contribution of the individual AUs, combined with the additional parameter configurations of the equity mechanism, does not have a considerable impact on the Theil index over time and, therefore, on the AUC. However, the findings reveal that all analyzed configurations have a lower AUC compared to the baseline, albeit only marginal.

The two best-performing configurations (highlighted in the second column in Fig. 2), which were identified for the softmax-based strategy for calculating inequity weights, each achieve an AUC of 18.91. Both configurations used a temperature $T = 1$ and calculated inequity weights for advantaged and disadvantaged AUs. The only difference is that one configuration allowed a weight decrease below zero when applying the inequity weight, whereas the other did not. This could imply that the original weights of advantaged AUs were never modified to such an extent so that the adjusted weight would fall below zero.

3) Exponential Decay: A total of 90 different configurations were analyzed for the exponential decay strategy to calculate inequity weights, testing five different regularization terms, i.e., 0.05 to 0.25 in increments of 0.05. In addition, when the exponential decay strategy is combined with the softmax function, temperatures ranging from 0.2 to 1.0 in increments of 0.2 were tested. The findings indicate that the different configurations, regardless of whether using the softmax function or not, combined with different settings of the Equity Mechanism, have varying effects on the Theil index over time and, therefore, on the AUC.

The highest AUC (21.28) was computed for two configurations that did not apply the softmax function. These configurations calculated inequity weights for both advantaged and disadvantaged AUs and used a regularization term $\lambda = 0.15$. The only difference between the two worst-performing configurations is whether allowing a weight decrease below zero, indicating that this parameter was not a decisive factor. The two configurations that achieved the lowest AUC (18.07) when the softmax function was applied, also used a regularization term $\lambda = 0.15$. These configurations used a temperature $T = 1$, while the other settings matched those of the two worst-performing configurations.

The AUC for the best configuration (highlighted in the third column in Fig. 2) equals 15.74. For this configuration, a regularization term $\lambda = 0.25$ was used without the softmax function, and inequity weights were calculated exclusively for disadvantaged AUs.

4) Discussion: The various configurations for each inequity weight calculation strategy were evaluated with respect to their AUC. The AUC for each configuration was determined based on the calculated Theil index values using a rolling window over the last 20 regulation instances. Based on the lowest AUC

TABLE II
AREA UNDER THE CURVE (AUC) OVER 32 THEIL INDEX VALUES FOR THE BEST CONFIGURATION(S) OF EACH STRATEGY TO CALCULATE INEQUITY WEIGHTS AND THE BASELINE

Strategy	**AUC**
Multiplication	13.63
Softmax-Based	18.91
Exponential Decay	15.74
Baseline	19.06

values, we identified and described the best configurations. Table II summarizes the lowest AUC values determined for each inequity weight calculation strategy, along with the AUC value of the baseline.

The overall lowest AUC is 13.63 and was computed for the multiplication strategy when using a constant factor $m = 10^6$. For this configuration, inequity weights were calculated for both advantaged and disadvantaged AUs, and a weight decrease below zero was disallowed when applying them. The results show that the Theil index over time can be considerably lowered compared to the baseline, indicating an increase in equity among AUs over time.

In the following, the multiplication strategy with a constant factor $m = 10^6$ is further evaluated with regard to the cost of equity.

B. *Cost of Equity*

The cost of equity quantifies how emphasizing equity affects the quality of the flight lists found by the TTMS. In general, the cost of equity is computed for each optimization in the 31 regulations in which inequity weights were applied. We report the cost of equity for the configuration that achieved the lowest AUC among all tested configurations. In this configuration, inequity weights were computed for both advantaged and disadvantaged AUs using the multiplication strategy with a constant factor $m = 10^6$, while disallowing weight adjustments below zero. Table III shows the mean and standard deviation of the cost of equity in percentage points calculated for the regulation instances 21–51.

In general, the cost of equity is rather small across all regulations. This indicates that emphasizing equity does not have a considerable impact on the quality of the flight lists found by the TTMS. However, since the cost of equity is calculated based on the min-max normalized fitness values, the magnitude of the cost depends on the range between the minimum and maximum of the fitness values. In optimizations where a large number of flights were considered, this range was comparatively large.

Averaged over all optimizations across the 31 regulations, the mean cost of equity is -3.23×10^{-3} percentage points for the airport and -10.55×10^{-3} percentage points for the AUs. This suggests that, on average, emphasizing equity slightly reduces the fitness of both objectives. However, the reduction is on the order of 10^{-3} percentage points and therefore rather small.

VI. SUMMARY AND FUTURE WORK

In this paper, we extended the architecture of the TTMS developed within the industrial research project HARMONIC [7] by incorporating our Equity Mechanism. The TTMS allows for collaborative optimization of arrival flight lists based on the preferences submitted in form of weight maps by the airport and the AUs. Although stakeholders can benefit from collaborative optimization, long-term equity among AUs must be ensured. Therefore, we designed a configurable Equity Mechanism that adjusts the weight maps of the AUs for optimization using inequity weights and tested it in a controlled lab environment at Frequentis Headquarters in Vienna. In particular, three strategies to calculate inequity weights—namely, multiplication, softmax-based, and exponential decay—were specified and a total of 120 configurations were evaluated.

We used weight maps from 51 regulations collected during the two-month live trial (May and June 2025) at Zurich Airport. To evaluate a single configuration of a strategy to calculate inequity weights, we re-ran each optimization and measured equity among AUs using the Theil index over a rolling window of the prior 20 regulation instances. Our results show that with a proper configuration of the Equity Mechanism, the Theil index over time can be considerably lowered, indicating greater equity among AUs. In addition, the impact on the quality of the TTMS-generated flight lists when emphasizing equity, measured by the cost of equity, is rather small for both the airport and the AUs.

The results obtained with the current Equity Mechanism seem promising as a starting point for future work. However, the generalizability of the results may be limited, as the data used for the laboratory exercise were collected during a specific period, i.e., spring, at a single airport, i.e., Zurich. Consequently, the Equity Mechanism should be validated across different temporal and geographical settings. Future work will (i) explore whether additional strategies to calculate inequity weights can match or surpass the current performance, including fine-tuning for the specific use case; (ii) investigate how to integrate the Equity Mechanism into a privacy-preserving setup when the TTMS is hosted by an honest-but-curious platform provider; and (iii) experiment with varying rolling window sizes and assess whether metrics other than the Theil index could serve as the basis for calculating inequity weights. A potential concern regarding the Theil index is the fact that an AU with an average delay of zero and an AU at the overall mean receive the same inequity weight, since their contribution to inequity, by convention, equals zero. However, no inference can be drawn about the probability that an AU exhibits an average delay of zero over a defined period of regulations.

TABLE III

Mean and standard deviation (SD) of the cost of equity in percentage points for the Airspace Users (AUs) and the airport, rounded to four decimal places, for the 31 regulations (regulation instances 21–51) in which inequity weights were applied. The cost of equity was calculated for the configuration using the multiplication strategy with a constant factor $m = 10^6$, which achieved the lowest the area under curve (AUC) of all configurations

	Regulation Instance							
	21		**22**		**23**		**24**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-89.8438$\times10^{-3}$	-19.0656$\times10^{-3}$	-4.4213$\times10^{-3}$	-4.0948$\times10^{-3}$	-0.0157$\times10^{-3}$	-0.0113$\times10^{-3}$	-0.0029$\times10^{-3}$	-0.0011$\times10^{-3}$
SD	323.9209$\times10^{-3}$	68.7242$\times10^{-3}$	18.3196$\times10^{-3}$	17.0099$\times10^{-3}$	0.0135$\times10^{-3}$	0.0090$\times10^{-3}$	0.0052$\times10^{-3}$	0.0037$\times10^{-3}$
	25		**26**		**27**		**28**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.0026$\times10^{-3}$	-0.0002$\times10^{-3}$	-0.0115$\times10^{-3}$	-0.0009$\times10^{-3}$	12.7603$\times10^{-3}$	-9.0944$\times10^{-3}$	-0.0049$\times10^{-3}$	-0.0013$\times10^{-3}$
SD	0.0049$\times10^{-3}$	0.0007$\times10^{-3}$	0.0216$\times10^{-3}$	0.0053$\times10^{-3}$	49.4422$\times10^{-3}$	35.2166$\times10^{-3}$	0.0050$\times10^{-3}$	0.0016$\times10^{-3}$
	29		**30**		**31**		**32**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.0196$\times10^{-3}$	-0.0103$\times10^{-3}$	-0.0597$\times10^{-3}$	-0.0258$\times10^{-3}$	-37.2185$\times10^{-3}$	-1.1489$\times10^{-3}$	-69.5290$\times10^{-3}$	-13.8433$\times10^{-3}$
SD	0.0187$\times10^{-3}$	0.0128$\times10^{-3}$	0.0185$\times10^{-3}$	0.0105$\times10^{-3}$	166.2544$\times10^{-3}$	5.1008$\times10^{-3}$	277.7471$\times10^{-3}$	55.3400$\times10^{-3}$
	33		**34**		**35**		**36**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.2032$\times10^{-3}$	-0.0241$\times10^{-3}$	-0.0200$\times10^{-3}$	-0.0083$\times10^{-3}$	-0.0837$\times10^{-3}$	0.0032$\times10^{-3}$	-169.7442$\times10^{-3}$	-17.0617$\times10^{-3}$
SD	0.2282$\times10^{-3}$	0.0413$\times10^{-3}$	0.0172$\times10^{-3}$	0.0118$\times10^{-3}$	0.0670$\times10^{-3}$	0.0077$\times10^{-3}$	831.2886$\times10^{-3}$	83.4844$\times10^{-3}$
	37		**38**		**39**		**40**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.0199$\times10^{-3}$	-0.0028$\times10^{-3}$	2.5581$\times10^{-3}$	-16.3817$\times10^{-3}$	-0.0120$\times10^{-3}$	0.0002$\times10^{-3}$	-0.0518$\times10^{-3}$	-0.0090$\times10^{-3}$
SD	0.0261$\times10^{-3}$	0.0054$\times10^{-3}$	25.5195$\times10^{-3}$	55.4591$\times10^{-3}$	0.0238$\times10^{-3}$	0.0013$\times10^{-3}$	0.0430$\times10^{-3}$	0.0120$\times10^{-3}$
	41		**42**		**43**		**44**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.0008$\times10^{-3}$	-0.0005$\times10^{-3}$	-0.0173$\times10^{-3}$	-0.0014$\times10^{-3}$	-0.0745$\times10^{-3}$	-0.0008$\times10^{-3}$	-0.0197$\times10^{-3}$	-0.0069$\times10^{-3}$
SD	0.0017$\times10^{-3}$	0.0010$\times10^{-3}$	0.0221$\times10^{-3}$	0.0052$\times10^{-3}$	0.0713$\times10^{-3}$	0.0029$\times10^{-3}$	0.0225$\times10^{-3}$	0.0095$\times10^{-3}$
	45		**46**		**47**		**48**	
	AUs	Airport	AUs	Airport	AUs	Airport	AUs	Airport
Mean	-0.0030$\times10^{-3}$	0.0006$\times10^{-3}$	-0.0027$\times10^{-3}$	-0.0003$\times10^{-3}$	-0.1051$\times10^{-3}$	-0.0211$\times10^{-3}$	2.6371$\times10^{-3}$	-25.6004$\times10^{-3}$
SD	0.0030$\times10^{-3}$	0.0005$\times10^{-3}$	0.0063$\times10^{-3}$	0.0004$\times10^{-3}$	0.1852$\times10^{-3}$	0.0273$\times10^{-3}$	23.9944$\times10^{-3}$	90.9761$\times10^{-3}$
	49		**50**		**51**			
	AUs	Airport	AUs	Airport	AUs	Airport		
Mean	-0.0658$\times10^{-3}$	-0.0122$\times10^{-3}$	-0.0052$\times10^{-3}$	-0.0041$\times10^{-3}$	-0.0072$\times10^{-3}$	-0.0031$\times10^{-3}$		
SD	0.0685$\times10^{-3}$	0.0181$\times10^{-3}$	0.0071$\times10^{-3}$	0.0060$\times10^{-3}$	0.0136$\times10^{-3}$	0.0035$\times10^{-3}$		

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